

National 5

Homework EF18

1. Simplify

(a) $\frac{x^2 \times x^5}{x^{-1}}$

(b) x^3^{-2}

(c) $a^3 \times a^{-4}{}^0$

2. (a) Find the value of $x^{\frac{1}{2}}(x^{\frac{1}{2}} - x^{-\frac{1}{2}})$ when $x = 3$.

(b) Find the value of $(a^{\frac{3}{2}} - a^{\frac{1}{2}})(a^{\frac{3}{2}} + a^{\frac{1}{2}})$ when $a = 2$.

3. Without using a calculator or decimal approximations, explain why $\sqrt{151} > 5\sqrt{6}$.

4. Solve each of these equations/inequalities.

(a) $3(x-1) + 2(1+x) = 17$

(b) $5(3x-1) - (1-x) < 9$

(c) $16 - 3x \leq x + 5(1-2x)$

(d) $(x-3)^2 - (x-5)^2 \leq 0$

5. Calculate, without a calculator:

(a) $8 \cdot 5 - 1 \cdot 2 \times 6$

(b) $8 \cdot 4^2 - 1 \cdot 6^2$

(c) $15 \cdot 3 \div 0 \cdot 9$

6. It can be shown that $1^2 + 2^2 + 3^2 + \dots + n^2 = \frac{1}{6}n(n+1)(2n+1)$. Using this formula,

(a) (i) Calculate $1^2 + 2^2 + 3^2 + \dots + 10^2$.

(ii) Calculate $11^2 + 12^2 + 13^2 + \dots + 20^2$.

(b) Find a formula for $1^2 + 2^2 + 3^2 + \dots + (2n)^2$.

7. The cost of hiring a taxi is as follows:

Standing charge: £3.

First 3 km: 80 pence per km.

Remainder of journey: 60 pence per km.

(i) Find the cost of a hire for a journey of 10 km.

(ii) Find a formula for the cost of a hire for a journey of t km. ($t > 3$).

8. Simplify each of the following.

(a) $a^4 \times a^{-3} \times a^{-1}$

(b) $\left(x^{\frac{1}{2}}\right)^6$

(c) $\frac{y^3 \times y^{-2}}{y^{-3}}$

(d) $(g^{-2})^{-4}$

(e) $(p^2 q^4)^{\frac{1}{2}}$

(f) $2c^2 \times 3c^{-3}$

(g) $10^{\frac{1}{2}} \times 10^{-\frac{2}{3}} \times 10^{\frac{1}{6}}$

9. (a) Solve $\frac{x}{3} = \frac{(1-x)}{5}$

(b) Express $\frac{4(2a-3)}{4a^2-9}$ in its simplest form.

10. (a) By "completing the square", show that $x^2 - 4x + 9 \geq 5$ for all real x .

(b) At what value of x does the minimum occur?

11. Factorise:

(a) $d^2 - 2d + 1$

(b) $2x^2 + 4x + 2$

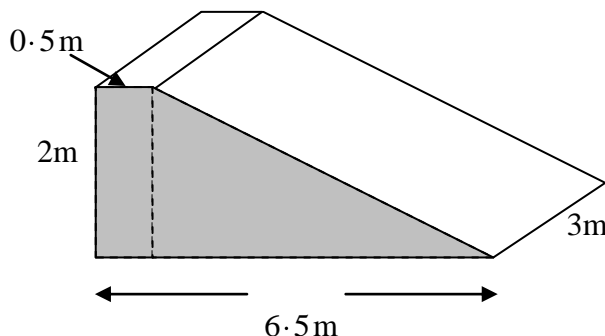
(c) $9m^2 - 16n^2$

(d) $2a^2 - 7a + 3$

(e) $a^2 - 5ab + 6b^2$

(f) $8x^2 - 2x - 3$

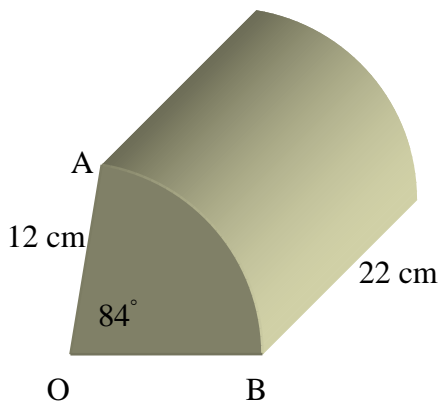
12. Find the volume of the solid sketched below. Give your answer in litres.



13. The cross-section of the prism sketched below is a sector of a circle of radius 12 cm.

The prism has length 22 cm and $\angle AOB = 84^\circ$.

Calculate the volume of the prism, correct to three significant figures.



14. A tank contains 180 litres of water.

The water flows out at a constant rate of 30 litres per minute.

(a) Draw an accurate graph of the volume V , of water in the tank against the time t , in minutes.

(b) How long does it take for the volume to fall to 135 litres?